# Dyadic Procedure for Planewave Scattering by Simply Moving, Electrically Small, Bianisotropic Spheres

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Z. Naturforsch. **46a**, 1033–1036 (1991); received February 13, 1991

A dyadic procedure is given for the planewave scattering response of an electrically small, bianisotropic sphere moving with a constant velocity in free space. Lorentz transformations and the polarizability dyadics of a stationary bianisotropic sphere are used in this procedure.

## 1. Introduction and Preliminaries

Bianisotropic media, characterized in the frequency ($\omega$) domain by the constitutive relations [1]

$$\boldsymbol{D}(\boldsymbol{r},\omega)=\varepsilon_0\,[\boldsymbol{\varepsilon}_{\rm r}(\omega)\cdot\boldsymbol{E}(\boldsymbol{r},\omega)+\boldsymbol{\zeta}(\omega)\cdot\boldsymbol{H}(\boldsymbol{r},\omega)]\ * \qquad (1\,\mathrm{a})$$

$$\boldsymbol{B}(\boldsymbol{r},\omega)=\mu_0\,[\boldsymbol{\xi}(\omega)\cdot\boldsymbol{E}(\boldsymbol{r},\omega)+\boldsymbol{\mu}_{\rm r}(\omega)\cdot\boldsymbol{H}(\boldsymbol{r},\omega)] \qquad (1\,\mathrm{b})$$

are not difficult to find in nature. Here, $\varepsilon_0$ and $\mu_0$ are the constitutive parameters of free space, $\boldsymbol{\varepsilon}_{\rm r}(\omega)$ is the relative permittivity dyadic, $\boldsymbol{\mu}_{\rm r}(\omega)$ is the relative permeability dyadic, while $\boldsymbol{\zeta}$ and $\boldsymbol{\xi}$ represent the magnetoelectric dyadics. Materials with dyadic permittivity ($\boldsymbol{\mu}_{\rm r}=\mu_{\rm r}\boldsymbol{I}$, $\boldsymbol{\zeta}=\boldsymbol{\xi}=0$) abound as crystals [2] and magnetoplasmas [3]; here and hereafter, $\boldsymbol{I}$ is the identity dyadic. Ferrites [1, 3] exhibit the magnetic Faraday effect ($\boldsymbol{\varepsilon}_{\rm r}=\varepsilon_{\rm r}\boldsymbol{I}$, $\boldsymbol{\zeta}=\boldsymbol{\xi}=0$). Natural optically active and biisotropic materials ($\boldsymbol{\varepsilon}_{\rm r}=\varepsilon_{\rm r}\boldsymbol{I}$, $\boldsymbol{\mu}_{\rm r}=\mu_{\rm r}\boldsymbol{I}$, $\boldsymbol{\zeta}=\zeta\boldsymbol{I}$, $\boldsymbol{\xi}=\xi\boldsymbol{I}$) are well-known to organic and physical chemists [4–6]. Even a simply moving, isotropic dielectric scatterer appears to be bianisotropic to a stationary observer [7]. Thus, a material characterized by (1 a, b) is the most general macroscopic, linear, non-diffusive electromagnetic substance. Physically realizable forms of the constitutive tensors have been discussed at length by Post [1] within the framework of Lorentz covariance [7].

Let a *stationary* sphere of radius $a$ made of a bianisotropic medium be embedded in free space; it is assumed that the origin $\boldsymbol{r}=\boldsymbol{0}$ is at the center of the sphere. Provided $a$ is small compared with the principal wavelengths inside the sphere as well as in free space, it has been shown in [8] that the scattering response of the stationary bianisotropic sphere can be computed in terms of an electric dipole moment $\boldsymbol{p}$ and a magnetic dipole moment $\boldsymbol{m}$. These *equivalent* dipole moments are given by

$$\boldsymbol{p}=\boldsymbol{\alpha}_{\rm ee}(\omega)\cdot\boldsymbol{E}_{\rm inc}(\boldsymbol{0})+\boldsymbol{\alpha}_{\rm em}(\omega)\cdot\boldsymbol{B}_{\rm inc}(\boldsymbol{0}), \qquad (2\,\mathrm{a})$$

$$\boldsymbol{m}=\boldsymbol{\alpha}_{\rm me}(\omega)\cdot\boldsymbol{E}_{\rm inc}(\boldsymbol{0})+\boldsymbol{\alpha}_{\rm mm}(\omega)\cdot\boldsymbol{B}_{\rm inc}(\boldsymbol{0}), \qquad (2\,\mathrm{b})$$

assuming a time-harmonic dependence $\exp[-i\omega t]$, with $\{\boldsymbol{E}_{\rm inc},\boldsymbol{B}_{\rm inc}\}$ representing the electromagnetic field incident on the sphere. The four polarizability dyadics for the small bianisotropic sphere are given by [8]

$$\boldsymbol{\alpha}_{\rm ee}(\omega)=4\pi a^3\,\varepsilon_0(\boldsymbol{\varepsilon}_{\rm r}+2\boldsymbol{I})^{-1}\cdot[(\boldsymbol{\varepsilon}_{\rm r}-\boldsymbol{I})+3\boldsymbol{\zeta}\cdot\boldsymbol{\Delta}_{\rm e}^{-1}\cdot\boldsymbol{\zeta}^{-1}], \qquad (3\,\mathrm{a})$$

$$\boldsymbol{\alpha}_{\rm em}(\omega)=-12\pi a^3\,\mu_0^{-1}\,\varepsilon_0\,\boldsymbol{\Delta}_{\rm h}^{-1}\cdot\boldsymbol{\xi}^{-1}, \qquad (3\,\mathrm{b})$$

$$\boldsymbol{\alpha}_{\rm me}(\omega)=-12\pi a^3\,\mu_0\,\boldsymbol{\Delta}_{\rm e}^{-1}\cdot\boldsymbol{\zeta}^{-1}, \qquad (3\,\mathrm{c})$$

$$\boldsymbol{\alpha}_{\rm mm}(\omega)=4\pi a^3(\boldsymbol{\mu}_{\rm r}+2\boldsymbol{I})^{-1}\cdot[(\boldsymbol{\mu}_{\rm r}-\boldsymbol{I})+3\boldsymbol{\xi}\cdot\boldsymbol{\Delta}_{\rm h}^{-1}\cdot\boldsymbol{\xi}^{-1}]. \qquad (3\,\mathrm{d})$$

On the right hand sides of (3 a–d), the argument $\omega$ has been suppressed; further,

$$\boldsymbol{\Delta}_{\rm e}(\omega)=\boldsymbol{I}-\boldsymbol{\zeta}^{-1}\cdot(\boldsymbol{\varepsilon}_{\rm r}+2\boldsymbol{I})\cdot\boldsymbol{\xi}^{-1}\cdot(\boldsymbol{\mu}_{\rm r}+2\boldsymbol{I}), \qquad (4\,\mathrm{a})$$

$$\boldsymbol{\Delta}_{\rm h}(\omega)=\boldsymbol{I}-\boldsymbol{\xi}^{-1}\cdot(\boldsymbol{\mu}_{\rm r}+2\boldsymbol{I})\cdot\boldsymbol{\zeta}^{-1}\cdot(\boldsymbol{\varepsilon}_{\rm r}+2\boldsymbol{I}), \qquad (4\,\mathrm{b})$$

while $\boldsymbol{\xi}^{-1}$ is the dyadic inverse of $\boldsymbol{\xi}$, etc. It is assumed here and hereafter that all dyadic inverses exist [1 (Chapts. 6 and 8)] or can be satisfactorily taken into account.

Expressions (3 a–d) constitute the most general result for a stationary sphere embedded in free space. It is the objective of this communication to extend these results by considering an electrically small bianisotropic sphere that is *moving* in free space with a constant velocity with respect to a *stationary* observer.

* The bold letters $\boldsymbol{\varepsilon\,\zeta\,\xi\,\mu\,I\,\alpha\,\Delta\,P\,M\,V\,G\,D}$ [in Eq. (19) and (20)] are Diadics, the bold letters $\boldsymbol{D\,r\,E\,H\,p\,B\,r\,u\,v\,A\,m}\,\nabla$ are vectors.

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The motivation for this work is twofold. First, the presented approach considerably generalizes previously reported work on scattering by simply moving chiral spheres [9], and may be useful for light scattering by complex aerosols. The second reason for this work is the potential use of these analyses in molecular dynamics [10], and the concurrent use in laser-based pump/probe spectroscopies for studying molecular gases [11, 12]; in this latter case, we may know the polarizability dyadics of (2 a, b) from other considerations [13, 14].

## 2. Analysis

Consider the four-dimensional *stationary frame* $K:(\boldsymbol{r}, t)$ in which all measurements are to take place. Rigidly attached to the center of the moving bianisotropic sphere is another coordinate system $K':(\boldsymbol{r}', t')$; the *moving frame* $K'$ moves with a velocity $\boldsymbol{v}$ with respect to $K$. It is assumed that at time $t = t' = 0$, the two systems $K$ and $K'$ coincide exactly. Then, the Lorentz transformations [15, 16]

$$\boldsymbol{r}' = \boldsymbol{r} + [(\gamma - 1)(\boldsymbol{r} \cdot \boldsymbol{u}_v) - \gamma v t]\, \boldsymbol{u}_v, \quad t' = (\gamma/c)[c t - \beta (\boldsymbol{r} \cdot \boldsymbol{u}_v)], \tag{5a, b}$$

hold with

$$\beta = v/c, \quad c = (\varepsilon_0 \mu_0)^{-1/2}, \quad \gamma = (1 - \beta^2)^{-1/2}, \quad v = |\boldsymbol{v}|, \quad \boldsymbol{u}_v = \boldsymbol{v}/v. \tag{6}$$

The corresponding transformations for the electric and the magnetic fields are given by [15, 16]

$$\boldsymbol{E}' = \gamma [\boldsymbol{E} + v (\boldsymbol{u}_v \times \boldsymbol{B})] - (\gamma - 1)(\boldsymbol{E} \cdot \boldsymbol{u}_v)\, \boldsymbol{u}_v, \tag{7a}$$

$$\boldsymbol{B}' = \gamma [\boldsymbol{B} - \beta (\boldsymbol{u}_v \times \boldsymbol{E})/c] - (\gamma - 1)(\boldsymbol{B} \cdot \boldsymbol{u}_v)\, \boldsymbol{u}_v, \tag{7b}$$

where $\boldsymbol{E} \equiv \boldsymbol{E}(\boldsymbol{r}, t)$, $\boldsymbol{E}' \equiv \boldsymbol{E}'(\boldsymbol{r}', t')$, etc.

### 2.1 *Incident Plane Wave in K*

The incident field $\{\boldsymbol{E}_{\text{inc}}, \boldsymbol{B}_{\text{inc}}\}$ in $K$ can be arbitrary so long as (i) its stationary source is never on or inside the scatterer, and (ii) its spectral content, as measured in $K'$ (see below), does not invalidate the applicability conditions of the long-wavelength approximation [8] inherent in the derivation of (2) and (3). Indeed, it may be convenient to work with the spatial and temporal Fourier transforms of $\boldsymbol{E}_{\text{inc}}(\boldsymbol{r}, t)$ and $\boldsymbol{B}_{\text{inc}}(\boldsymbol{r}, t)$; i.e., with a planewave spectral decomposition [17] of $\boldsymbol{E}_{\text{inc}}(\boldsymbol{r}, t)$ and $\boldsymbol{B}_{\text{inc}}(\boldsymbol{r}, t)$.

Therefore, without any particular loss of generality, the incident field in $K$ is assumed to be the plane wave

$$\boldsymbol{E}_{\text{inc}}(\boldsymbol{r}, t) = \boldsymbol{A} \exp [i (k\, \boldsymbol{u}_{\text{inc}} \cdot \boldsymbol{r} - \omega t)], \tag{8a}$$

$$\boldsymbol{B}_{\text{inc}}(\boldsymbol{r}, t) = (\boldsymbol{u}_{\text{inc}} \times \boldsymbol{A}/c) \exp [i (k\, \boldsymbol{u}_{\text{inc}} \cdot \boldsymbol{r} - \omega t)], \tag{8b}$$

where $\boldsymbol{u}_{\text{inc}}$ is a unit vector parallel to the direction of propagation, while

$$\boldsymbol{u}_{\text{inc}} \cdot \boldsymbol{A} \equiv 0, \quad k = \omega/c. \tag{8c, d}$$

### 2.2 *Incident Plane Wave in K′*

The field transformations (7 a, b) can be used to obtain the incident field in the moving frame $K'$. The incident field in $K'$ is still a transverse planewave specified by

$$\boldsymbol{E}'_{\text{inc}}(\boldsymbol{r}', t') = \boldsymbol{A}' \exp [i (k'\, \boldsymbol{u}'_{\text{inc}} \cdot \boldsymbol{r}' - \omega' t')], \tag{9a}$$

$$\boldsymbol{B}'_{\text{inc}}(\boldsymbol{r}', t') = (\boldsymbol{u}'_{\text{inc}} \times \boldsymbol{A}'/c) \exp [i (k'\, \boldsymbol{u}'_{\text{inc}} \cdot \boldsymbol{r}' - \omega' t')], \tag{9b}$$

which expressions are in consonance with [15]. In these expressions and hereafter,

$$\omega' = \omega \gamma (1 - \beta\, \boldsymbol{u}_v \cdot \boldsymbol{u}_{\text{inc}}), \quad k' = \omega'/c, \tag{10a, b}$$

$$\boldsymbol{u}'_{\text{inc}} = (\omega/\omega')[\boldsymbol{u}_{\text{inc}} + (\gamma - 1)(\boldsymbol{u}_{\text{inc}} \cdot \boldsymbol{u}_v)\, \boldsymbol{u}_v - \gamma \beta\, \boldsymbol{u}_v], \tag{10c}$$

$$\boldsymbol{A}' = (\omega'/\omega)\, \boldsymbol{A} + [\gamma \beta\, \boldsymbol{u}_{\text{inc}} - (\gamma - 1)\, \boldsymbol{u}_v](\boldsymbol{A} \cdot \boldsymbol{u}_v). \tag{10d}$$

### 2.3 *Scattered Field in K′*

As mentioned earlier, the frequency $\omega'$ is such that the long-wavelength approximation is to hold in $K'$; furthermore, free space is Lorentz-invariant [1]. This means that in $K'$, the scatterer is equivalent to the dipoles $\boldsymbol{p}'$ and $\boldsymbol{m}'$ oscillating at a frequency $\omega'$ given by

$$\boldsymbol{p}' = [\boldsymbol{\alpha}_{\text{ee}}(\omega') + (1/c)\, \boldsymbol{\alpha}_{\text{em}}(\omega') \cdot (\boldsymbol{u}'_{\text{inc}} \times \boldsymbol{I})] \cdot \boldsymbol{A}', \tag{11a}$$

$$\boldsymbol{m}' = [\boldsymbol{\alpha}_{\text{me}}(\omega') + (1/c)\, \boldsymbol{\alpha}_{\text{mm}}(\omega') \cdot (\boldsymbol{u}'_{\text{inc}} \times \boldsymbol{I})] \cdot \boldsymbol{A}'. \tag{11b}$$

After making use of (10 a–d), (11 a, b) can be rewritten as

$$\boldsymbol{p}' = \boldsymbol{P} \cdot \boldsymbol{A}, \quad \boldsymbol{m}' = \boldsymbol{M} \cdot \boldsymbol{A}, \tag{12a, b}$$

where the two dyadics, $\boldsymbol{P}$ and $\boldsymbol{M}$, are defined as

$$\begin{aligned} \boldsymbol{P} = \boldsymbol{\alpha}_{\text{ee}}(\omega') \cdot \{&\gamma (1 - \beta\, \boldsymbol{u}_v \cdot \boldsymbol{u}_{\text{inc}})\, \boldsymbol{I} \\ &+ \gamma \beta\, \boldsymbol{u}_{\text{inc}} \boldsymbol{u}_v - (\gamma - 1)\, \boldsymbol{u}_v \boldsymbol{u}_v\} \\ &+ (1/c)\, \boldsymbol{\alpha}_{\text{em}}(\omega') \cdot \{\boldsymbol{u}_{\text{inc}} \times \boldsymbol{I} \\ &+ (\gamma - 1)(\boldsymbol{u}_v \cdot \boldsymbol{u}_{\text{inc}})\, \boldsymbol{V} - \gamma \beta\, \boldsymbol{V} - (\gamma - 1)\, \boldsymbol{V} \cdot \boldsymbol{u}_{\text{inc}} \boldsymbol{u}_v\}, \end{aligned} \tag{13a}$$

$$M = \boldsymbol{\alpha}_{me}(\omega') \cdot \{\gamma(1-\beta\, \boldsymbol{u}_v \cdot \boldsymbol{u}_{inc}) \boldsymbol{I} + \gamma\beta\, \boldsymbol{u}_{inc} \boldsymbol{u}_v - (\gamma-1)\, \boldsymbol{u}_v \boldsymbol{u}_v\} + (1/c)\, \boldsymbol{\alpha}_{mm}(\omega') \cdot \{\boldsymbol{u}_{inc} \times \boldsymbol{I} + (\gamma-1)(\boldsymbol{u}_v \cdot \boldsymbol{u}_{inc})\, \boldsymbol{V} - \gamma\beta\, \boldsymbol{V} - (\gamma-1)\, \boldsymbol{V} \cdot \boldsymbol{u}_{inc} \boldsymbol{u}_v\}, \tag{13b}$$

with

$$\boldsymbol{V} = \boldsymbol{u}_v \times \boldsymbol{I}. \tag{14}$$

Using (2a, b), the scattered field in $K'$ can be obtained from the dipole moments as [8]

$$\boldsymbol{E}'_{sc}(\boldsymbol{r}', t') = (\omega'^2 \mu_0\, \boldsymbol{G}(\boldsymbol{r}') \cdot \boldsymbol{p}' + i\omega' [\nabla' \times \boldsymbol{G}(\boldsymbol{r}')] \cdot \boldsymbol{m}') \cdot \exp[-i\omega' t'], \tag{15a}$$

$$\boldsymbol{B}'_{sc}(\boldsymbol{r}', t') = \mu_0 (\omega'^2 \varepsilon_0\, \boldsymbol{G}(\boldsymbol{r}') \cdot \boldsymbol{m}' - i\omega' [\nabla' \times \boldsymbol{G}(\boldsymbol{r}')] \cdot \boldsymbol{p}') \cdot \exp[-i\omega' t'], \tag{15b}$$

with

$$\boldsymbol{G}(\boldsymbol{r}') = (\boldsymbol{I} + \nabla' \nabla'/k'^2)\{\exp[i k' r']/4\pi r'\}. \tag{16}$$

Since the near zone in $K'$ is small due to the small electrical size of the scatterer [18], the scattered field (15a, b) in $K'$ may be conveniently approximated as

$$\boldsymbol{E}'_{sc}(\boldsymbol{r}', t') \cong -(k'^2 \exp[i(k' r' - \omega' t')]/4\pi r'^3) \cdot [\varepsilon_0^{-1} (\boldsymbol{r}' \times \boldsymbol{I}) \cdot (\boldsymbol{r}' \times \boldsymbol{I}) \cdot \boldsymbol{p}' + c\, r' (\boldsymbol{r}' \times \boldsymbol{I}) \cdot \boldsymbol{m}'], \tag{17a}$$

$$\boldsymbol{B}'_{sc}(\boldsymbol{r}', t') \cong -(k'^2 \exp[i(k' r' - \omega' t')]/4\pi r'^3) \cdot [(\boldsymbol{r}' \times \boldsymbol{I}) \cdot (\boldsymbol{r}' \times \boldsymbol{I}) \cdot \boldsymbol{m}' - \eta_0 r' (\boldsymbol{r}' \times \boldsymbol{I}) \cdot \boldsymbol{p}'] \tag{17b}$$

with $\eta_0 = \sqrt{(\mu_0/\varepsilon_0)}$ being the intrinsic impedance of free space.

### 2.4 Scattered Field in K

To calculate $\boldsymbol{E}_{sc}(\boldsymbol{r}, t)$ from (17a, b) the Lorentz field transformations (7a, b) have to be applied to (17a, b) in reverse. This leads to some tedious algebra. Hence, it is convenient to define the following functions:

$$R(\boldsymbol{r}, t) = +\sqrt{[r^2 + \gamma^2 (v t - \boldsymbol{r} \cdot \boldsymbol{u}_v)^2 - (\boldsymbol{r} \cdot \boldsymbol{u}_v)^2]}, \tag{18a}$$

$$\Phi(\boldsymbol{r}, t) = -[k^2 \gamma^2 (1 - \beta\, \boldsymbol{u}_v \cdot \boldsymbol{u}_{inc})^2 / 4\pi R^3] \cdot \exp[i k \gamma (1 - \beta\, \boldsymbol{u}_v \cdot \boldsymbol{u}_{inc})(R - \gamma c t + \gamma\beta\, \boldsymbol{r} \cdot \boldsymbol{u}_v)], \tag{18b}$$

$$\boldsymbol{D}(\boldsymbol{r}, t) = \boldsymbol{r} \times \boldsymbol{I} + [(\gamma-1)(\boldsymbol{r} \cdot \boldsymbol{u}_v) - \gamma v t]\, \boldsymbol{V}. \tag{18c}$$

Then, the inversion process leads to

$$\boldsymbol{E}_{sc}(\boldsymbol{r}, t)/\Phi(\boldsymbol{r}, t) = \varepsilon_0^{-1} \{-\gamma \boldsymbol{V} \cdot (\boldsymbol{V} \cdot \boldsymbol{D} - \beta R \boldsymbol{I}) + \boldsymbol{u}_v (\boldsymbol{u}_v \cdot \boldsymbol{D})\} \cdot \boldsymbol{D} \cdot \boldsymbol{p}' + c\{-\gamma \boldsymbol{V} \cdot (R \boldsymbol{V} + \beta \boldsymbol{D}) + R\, \boldsymbol{u}_v \boldsymbol{u}_v\} \cdot \boldsymbol{D} \cdot \boldsymbol{m}' \tag{19}$$

Using the dyadics $\boldsymbol{P}$ and $\boldsymbol{M}$ defined in (12a, b) and (13a, b), (19) reduces to

$$\boldsymbol{E}_{sc}(\boldsymbol{r}, t) = \Phi(\boldsymbol{r}, t) \cdot [\varepsilon_0^{-1} \{-\gamma \boldsymbol{V} \cdot (\boldsymbol{V} \cdot \boldsymbol{D} - \beta R \boldsymbol{I}) + \boldsymbol{u}_v (\boldsymbol{u}_v \cdot \boldsymbol{D})\} \cdot \boldsymbol{D} \cdot \boldsymbol{P} + c\{-\gamma \boldsymbol{V} \cdot (R \boldsymbol{V} + \beta \boldsymbol{D}) + R\, \boldsymbol{u}_v \boldsymbol{u}_v\} \cdot \boldsymbol{D} \cdot \boldsymbol{M}] \cdot \boldsymbol{A}. \tag{20}$$

Finally, the scattered magnetic field $\boldsymbol{B}_{sc}(\boldsymbol{r}, t)$ in $K$ can be readily derived using the Faraday-Maxwell relation

$$\boldsymbol{B}_{sc}(\boldsymbol{r}, t) = (1/c r)\, \boldsymbol{r} \times \boldsymbol{E}_{sc}(\boldsymbol{r}, t) = (1/c r)(\boldsymbol{r} \times \boldsymbol{I}) \cdot \boldsymbol{E}_{sc}(\boldsymbol{r}, t) \tag{21}$$

along with (20).

As a check, $v = 0$ is set in (19); then $\omega = \omega'$, $k = k'$, $\boldsymbol{r}' = \boldsymbol{r}$, $t' = t$, $\boldsymbol{A} = \boldsymbol{A}'$ and $\boldsymbol{u}'_{inc} = \boldsymbol{u}_{inc}$ from (5a, b) and (10a–d). Equation (19) reduces to

$$\boldsymbol{E}_{sc}(\boldsymbol{r}, t) \cong -(k^2 \exp[i(k r - \omega t)]/4\pi r^3) \cdot [\varepsilon_0^{-1} (\boldsymbol{r} \times \boldsymbol{I}) \cdot (\boldsymbol{r} \times \boldsymbol{I}) \cdot \boldsymbol{P} + c\, r (\boldsymbol{r} \times \boldsymbol{I}) \cdot \boldsymbol{M}] \cdot \boldsymbol{A}, \tag{22a}$$

that is the same as (17a); at the same time, the dyadics $\boldsymbol{P}$ and $\boldsymbol{M}$ reduce to

$$\boldsymbol{P} = \boldsymbol{\alpha}_{ee}(\omega) + (1/c)\, \boldsymbol{\alpha}_{em}(\omega) \cdot \{\boldsymbol{u}_{inc} \times \boldsymbol{I}\}, \tag{22b}$$

$$\boldsymbol{M} = \boldsymbol{\alpha}_{me}(\omega) + (1/c)\, \boldsymbol{\alpha}_{mm}(\omega) \cdot \{\boldsymbol{u}_{inc} \times \boldsymbol{I}\}. \tag{22c}$$

Thus the results of [8] for a *stationary*, electrically small, bianisotropic sphere are identically recovered.

## 3. Concluding Remarks

A dyadic procedure has been given for the planewave scattering response of an electrically small, bianisotropic sphere moving with a constant velocity in free space. Lorentz transformations and the polarizability dyadics of a stationary bianisotropic sphere have been used in this procedure. Specific simplifications of the presented approach are expected to be useful for electromagnetic scattering studies on aerosols and gases. Since the dyadics involved can be interpreted in terms of $3 \times 3$ matrices, the presented method is computationally tractable. Furthermore, dyadic analysis has permitted the use of an arbitrary velocity vector $v\, \boldsymbol{u}_v$ and incident plane wave propagation direction $\boldsymbol{u}_{inc}$.

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